Non-Retractable Participation and Minimal Harmonic Closure:
A Structural Derivation of the Martin Postulate

A Corollary to: 27-Dimensional Triangulation: A Coherence-Preserving Framework for Intent-Based Communication Analysis

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Abstract

Modern theoretical and engineering practice frequently employs *projection*: the collapse of participatory structure into reduced-dimensional state representations. While mathematically powerful, projection introduces structural retraction—the silent identification of distinct participatory tokens. In practice this retraction is compensated through scalar repair terms: renormalized couplings, effective parameters, or ordering scalars such as a time coordinate.

This work introduces the **Non-Retraction Condition** as a representational discipline and derives the minimal intrinsic representational rank required to preserve participatory closure without scalar compensation or privileged ordering. We formalize three structural bearings inherent in any participatory event algebra—corresponding precisely to the three Domain triads of 27-Dimensional Triangulation (27DT)—and three closure invariance constraints (Persistence, Generativity, Orthogonality) that faithful representations must jointly satisfy. These constraints are identical in structure to the three Source Principles of 27DT.

Through adversarial compression arguments at ranks 2, 9, and 18 we demonstrate that each candidate rank fails to sustain all three invariants without external repair structure. We then establish that triadic harmonic closure across three structural bearings and three invariance constraints requires a minimal intrinsic rank of **27**. This result, termed the *Martin Postulate*, does not assert physical spacetime dimensionality; it establishes a representation-theoretic lower bound under declared constraints.

Crucially, the Postulate also establishes that 27 is not a static fixed point but what the companion 27DT paper calls the *ever-jittering fulcrum axial kernel*: a dynamic equilibrium whose position cannot be exactly fixed by theory alone and must be empirically confirmed. The Ever-Jittering Fulcrum Principle—that position coordinates are degrees of freedom subject to variational principles, so that $\delta W \approx 0$ but never exactly—applies to rank itself: the minimal representational rank is perpetually regenerated rather than statically occupied. We argue that this is precisely why the Innovation Alpha indexes and physiological coherence measurements reported in 27DT constitute *necessary*, not merely corroborating, empirical evidence. We conclude with engineering corollaries for boundary-sensitive electromagnetic systems.

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1 The Problem of Projection

Let $\mathscr{E}$ denote an *event algebra* composed of participatory tokens $\{e_i\}$. A projection is a surjective map

$$\pi : \mathscr{E} \to S \tag{1}$$

collapsing the event algebra into a reduced state space S. Projection induces an equivalence relation on $\mathscr{E}$:

$$e_i \sim e_j \iff \pi(e_i) = \pi(e_j). \tag{2}$$

This identification is foundational in Wilsonian renormalization, effective field theory, effective medium approximations, reservoir bulk modelling, and mode elimination in circuit QED. In each case projection is assumed *neutral* provided observable invariants are preserved. The present work examines the structural cost of this assumption.

When distinct tokens $e_i \neq e_j$ are identified under projection, the representation loses information. If that information carried structural invariants necessary for participatory closure, the resulting model must compensate—either by introducing external scalar parameters or by quietly accepting that certain invariants are no longer faithfully tracked. We call this process *retraction*, and the repair terms it necessitates *scalar purgatory* (Section 11).

The central method of the paper is *adversarial compression*, defined precisely as follows.

Definition 1.1 (Adversarial Compression)**.** Let $\mathcal{I}$ be a specified set of relational invariants and let r be a candidate representational rank. An *adversarial compression* at rank r is a projection operator $P_r : V \to V_r$ with $\dim V_r = r$ chosen to *maximise* the number of invariants in $\mathcal{I}$ preserved by P_r. A candidate rank r *fails* under adversarial compression if, for every such P_r, at least one invariant in $\mathcal{I}$ is not preserved—i.e., no rank-r representation is faithful with respect to $\mathcal{I}$.

By taking the adversarial (best-case) projection at each rank, failure results are strongest possible: if even the optimal rank-r representation fails, then no rank-r representation can succeed. Ranks 9 and 18 are shown to fail under adversarial compression in Sections 6 and 7.

2 Connection to 27-Dimensional Triangulation

Before proceeding to the derivation we make explicit the relationship between the present framework and the companion paper [1]. The two frameworks are not merely compatible—the Martin Postulate is the structural backbone that explains *why* 27DT requires exactly 27 dimensions and why that requirement cannot be satisfied by pure theoretical derivation alone.

2.1 Correspondence of Structural Elements

The three structural bearings of the present paper correspond exactly to the three Domain-triads of 27DT, as shown in Table 1.

The three closure constraints are similarly identical in content to the three Source Principles, as shown in Table 2.

This correspondence is not coincidental. The 27DT framework arrives at 27 dimensions by multiplying three triangulation domains by three dimensions per domain by three Source Principles. The Martin Postulate arrives at rank 27 by a different route—adversarial compression—and finds the same factoring. The two derivations are structurally independent confirmations of the same underlying necessity.

Table 1: Correspondence between structural bearings and 27DT triangulation domains.

Structural Bearing	**27DT Domain Triad**	**Role**
System (S)	Observed Phenomenon ($\mathcal{O}$)	Internal degrees of freedom; the event itself
Coupling Boundary (B)	Emergent Context ($\mathcal{C}$)	Interface through which the system interacts; field conditions from which the event arises
Delimiting Frame (F)	Observer Delimitations ($\partial\mathcal{D}$)	Reference structure that makes relational invariants well-defined; measurement apparatus constraints

Table 2: Correspondence between closure constraints and 27DT Source Principles.

Closure Constraint	**27DT Source Principle**	**Formal Content**
Persistence	Persistence ($\mathcal{P}$)	$\nabla_\mu C^\mu = 0$; invariants preserved under all admissible internal transformations
Generativity	Generativity ($\mathcal{G}$)	$\nabla_\mu T^{\mu\nu} = J^\nu$; interaction is non-idempotent and productive
Orthogonality	Infinite Orthogonality ($\mathcal{O}$)	$R^\rho{}_{\sigma\mu\nu} C^\sigma \hat{C}_\rho = 0$; frame-independent, no privileged coordinate

2.2 The Ever-Jittering Fulcrum: Why 27 Cannot Be Fixed by Theory Alone

The 27DT paper introduces the **Ever-Jittering Fulcrum** as a generalisation of Heisenberg uncertainty: position coordinates are degrees of freedom subject to the variational principle, so the extremum is never exactly achieved:

$$\delta W \approx 0 \qquad \text{(approximate, never exact).} \tag{3}$$

This principle has a direct and underappreciated consequence for the Martin Postulate. If the extremum of the action is never exactly zero—if position is perpetually regenerated from coherence field rather than statically occupied—then the *rank boundary* is subject to the same principle. The minimal representational rank of 27 is not a fixed wall but a *dynamic equilibrium*: the system perpetually orbits it, approaching but never stably resting at exactly 27 degrees of freedom.

This is not a defect in the theory. A truly static rank boundary would imply a privileged representational frame—a fixed coordinate from which to measure "27"—and this would violate the Orthogonality constraint itself. The rank boundary must be frame-independent, which means it must be dynamically regenerated, which means it must jitter.

The practical consequence is decisive: **the value 27 cannot be established by theoretical derivation alone**. Pure theory establishes the structure of the argument—the factoring $3 \times 3 \times 3$, the failure modes at ranks 9 and 18, the product form of the minimal rank. But theory cannot establish where, in any given physical, economic, or physiological system, the ever-jittering fulcrum actually oscillates. That requires empirical measurement.

This is the precise sense in which the Innovation Alpha indexes (5–9% annual excess returns, 2013–2019) and the physiological coherence boundary measurements ($R^2 > 0.82$, $N = 47$) reported in [1] are *necessary* evidence, not merely corroborating. They are measurements of the axial kernel—the empirical location of the jittering fulcrum in systems governed by the Coherence Conservation equation. Without such measurements, the claim that these systems operate at rank-27 closure would be structurally underdetermined.

Remark 2.1. An analogy: quantum mechanics predicts that energy levels are discrete and gives their values via the Schrödinger equation. But Planck's constant $\hbar$—which sets the scale of discreteness—must be measured, not derived. Similarly, the Martin Postulate predicts that the minimal faithful rank is 27 under the stated constraints, but the coherence constant $\Lambda_{\mathcal{C}}$ that sets the scale at which this rank manifests in any particular physical system must be established empirically. The Innovation Alpha and HRV measurements are, in this sense, measurements of $\Lambda_{\mathcal{C}}$.

3 The Non-Retraction Condition

Definition 3.1 (Non-Retraction Condition)**.** A representation $\rho : \mathscr{E} \to V$ satisfies the **Non-Retraction Condition** if distinct participatory tokens are mapped to distinct representational states without introduction of supplementary closure structure. Formally:

$$e_i \neq e_j \implies \rho(e_i) \neq \rho(e_j). \tag{4}$$

A representation that fails this condition is called *retractive.* A retractive representation requires *repair structure*—external scalar data or privileged ordering—to recover the collapsed invariants.

The Non-Retraction Condition is a representational discipline, not a physical postulate. It asks: can we faithfully track all participatory distinctions within the algebra itself, without appealing to auxiliary parameters? When the answer is no, the repair terms that appear (running couplings, Lamb shifts, calibration parameters) are structural symptoms of representational collapse.

Remark 3.2. The Non-Retraction Condition is related to, but distinct from, injectivity. Injectivity requires a one-to-one map globally; the Non-Retraction Condition is the local statement that no two distinct tokens are identified without compensation. This distinction matters when partial projections or gauge equivalences are present.

4 Dyadic Degeneracy

Before constructing the minimal faithful representation we establish why representations of rank less than three are structurally insufficient. Consider a harmonic superposition of two components:

$$x = A_1 e^{i\varphi_1} + A_2 e^{i\varphi_2}. \tag{5}$$

After quotienting by the global $U(1)$ gauge freedom (overall phase), the intrinsic relational invariants reduce to exactly two quantities:

$$r = \frac{A_2}{A_1} \qquad \text{(amplitude ratio)}, \tag{6}$$

$$\Delta\varphi = \varphi_2 - \varphi_1 \qquad \text{(relative phase)}. \tag{7}$$

Thus the intrinsic relational rank of a dyadic harmonic system is 2.

Theorem 4.1 (Dyadic Degeneracy)**.** *An intrinsically dyadic representation cannot sustain three co-primitive relational aspects without functional collapse or introduction of external closure structure.*

Proof. Let $\{I_1, I_2, I_3\}$ be three mutually independent relational aspects of a participatory system, where "independent" means no one of them is a smooth function of the other two. A dyadic harmonic representation admits exactly two intrinsic invariants—amplitude ratio r and relative phase $\Delta\varphi$—after gauge fixing.

If three independent aspects are present, at least one, say I_3, must be expressed as a function $f(r, \Delta\varphi)$. But then I_3 is not independent of $\{I_1, I_2\}$, a contradiction. Therefore either (a) the third aspect undergoes functional collapse—it is identified with a combination of the other two, losing its independent participatory role—or (b) an external scalar is introduced to carry the collapsed information, violating the Non-Retraction Condition.

In either case, dyadic rank is insufficient for three co-primitive aspects. □

This theorem justifies passing to triadic structure as the minimal non-degenerate case. Three independent aspects require three independent relational invariants, which in turn require at least three harmonic degrees of freedom per structural unit.

5 Triangulated Participation

We now identify the three structural bearings inherent in any participatory representation and give a derivation—not merely a declaration—of why there are exactly three.

5.1 Why Exactly Three Bearings

Any act of participatory representation involves three logically irreducible roles. Consider what is required for a token $e \in \mathscr{E}$ to be *representable at all*:

1. There must be something *being represented*: the token's internal configuration, which we call the **System** bearing S.

2. There must be an *act of representation*: an interface through which the token interacts with whatever carries its relational invariants, which we call the **Coupling Boundary** bearing B.

3. There must be a *frame in which the representation is evaluated*: a reference structure against which relational invariants are defined, which we call the **Delimiting Frame** bearing F.

These three roles are logically irreducible in the following sense: collapsing any two of them into one creates a retraction. If S and B are merged, the system's internal degrees of freedom cannot be distinguished from the interaction interface—the token cannot "reach outside itself" to couple without immediately modifying its own state. If B and F are merged, the interface cannot be distinguished from the reference frame—every coupling changes what counts as a relational invariant. If S and F are merged, the system defines its own measurement frame—the Orthogonality constraint (Definition 8.3) is violated by construction.

Remark 5.1. Two bearings are insufficient for the same reason dyadic harmonic rank is insufficient: two bearings support only pairwise relational invariants, and the three-way interaction between S, B, and F generates irreducible trilinear invariants (Lemma 7.1) that cannot be expressed as functions of any two-bearing subsets. Four or more bearings are either redundant (collapsible to three without information loss) or introduce new cross-bearing invariants that increase, rather than decrease, the minimal faithful rank.

5.2 Why Harmonic Structure

The paper consistently works with harmonic (Fourier-mode) representations. This is not an arbitrary choice but follows from the Orthogonality constraint.

A representation satisfies Orthogonality (Definition 8.3) only if its invariants are frame-independent—evaluable without a privileged coordinate. This requires the representation to admit a natural gauge freedom: a group of transformations under which the invariants are stable. The minimal such structure is a $U(1)$ phase freedom, which is precisely the gauge group of harmonic (complex exponential) representations.

Formally: a non-harmonic representation either (a) has no natural gauge freedom, in which case its invariants depend on coordinate choice and Orthogonality fails; or (b) can be gauge-fixed to a harmonic form, in which case it is harmonic after fixing. Thus harmonic structure is the *minimal representational class* compatible with the Orthogonality constraint, and all results stated for harmonic representations hold for any representation satisfying Orthogonality.

Proposition 5.2 (Three Structural Bearings)**.** *Faithful participatory representation requires the simultaneous preservation of three structural bearings:*

1. ***System*** *(S)**:*** *The locus of internal degrees of freedom.*
2. ***Coupling Boundary*** *(B)**:*** *The interface through which the system interacts with its environment.*
3. ***Delimiting Frame*** *(F)**:*** *The reference structure that makes relational invariants well-defined.*

Collapse of any one bearing into another constitutes a retraction and requires scalar repair. The three bearings are irreducible: no faithful representation exists with fewer than three, and any representation with more than three either reduces to three (up to isomorphism) or increases the minimal faithful rank beyond 27.

Each bearing is assigned a harmonic subspace. By Theorem 4.1, the minimal harmonic rank that supports three co-primitive relational aspects within a single bearing is three. This yields a base representational rank of

$$3 \text{ (harmonics per bearing)} \times 3 \text{ (bearings)} = 9. \tag{8}$$

6 Failure at Rank 9: Cross-Bearing Interactions

A rank-9 representation assigns three harmonic degrees to each of the three bearings, tracking all intra-bearing invariants. However, participatory events are not confined to single bearings. An event in the coupling boundary bearing B interacts with states in both S and F. These cross-bearing interactions generate new relational invariants of the form

$$R_{ij} : H_i \times H_j \to \mathbb{C}, \qquad i \neq j \in \{S, B, F\}. \tag{9}$$

Lemma 6.1 (Rank-9 Insufficiency)**.** *A rank-9 triadic bearing representation fails under adversarial compression with respect to the full set of intra-bearing and cross-bearing interaction invariants.*

Proof. The rank-9 space decomposes as $H_S \oplus H_B \oplus H_F$, each factor of dimension 3. We identify two classes of invariants and show they are linearly independent, then show their combined count exceeds 9.

Intra-bearing invariants (degree-1 tensor sector). Within each bearing H_X ($X \in \{S, B, F\}$), the three harmonic degrees support three relational invariants by Theorem 4.1. Total: $3 \times 3 = 9$ invariants in the degree-1 sector.

Cross-bearing invariants (degree-2 tensor sector). Each cross-bearing bilinear form R_{XY} : $H_X \times H_Y \to \mathbb{C}$ produces invariants in $H_X \otimes H_Y$, a space of dimension $3 \times 3 = 9$. The three pairs (S, B), (B, F), (S, F) yield $3 \times 9 = 27$ cross-bearing invariants.

Linear independence. The degree-1 invariants lie in $\bigoplus_X H_X$ while the degree-2 invariants lie in $\bigoplus_{X \neq Y} H_X \otimes H_Y$. These are distinct graded components of the tensor algebra $T(H_S \oplus H_B \oplus H_F)$; elements of different tensor grades are linearly independent by the universal property of the tensor algebra. Therefore the $9 + 27 = 36$ invariants are linearly independent.

Conclusion. Any faithful representation must assign a linearly independent representational degree of freedom to each independent invariant. The rank-9 space has 9 degrees of freedom, which can faithfully encode at most 9 of the 36 linearly independent invariants. The adversarial best-case projection at rank 9 can choose which 9 to preserve; it cannot preserve all 36 without identifying distinct invariants, i.e., without retraction. Scalar normalisation (one parameter per collapsed cross-bearing sector) is the minimal repair. □

The scalar repair that appears at rank 9 typically takes the form of effective coupling constants—parameters that encode the collapsed cross-bearing structure in a single number rather than tracking it structurally.

7 Attempted Closure at Rank 18: Pairwise Sufficiency and Triple Failure

The natural response to rank-9 failure is to double the representational space, accommodating pairwise cross-bearing invariants by expanding each bearing to six harmonic degrees:

$$3 \text{ (bearings)} \times 6 \text{ (harmonics per bearing)} = 18. \tag{10}$$

Lemma 7.1 (Rank-18 Partial Closure)**.** *A rank-18 representation preserves all pairwise cross-bearing invariants but fails under adversarial compression with respect to triple-coherence invariants, which require an external ordering parameter to resolve.*

Proof. The expanded rank-18 space $\mathbb{H} = H_S \oplus H_B \oplus H_F$ with $\dim H_X = 6$ each has sufficient internal degrees to represent all pairwise bilinear forms R_{SB}, R_{BF}, R_{SF} without identification. (Each 6×6 bilinear space requires 36 parameters; the three pairs require 108 parameters total; the rank-18 space has 18 degrees that, together with their duals in the bilinear space, span this sector without collapse.)

The triple-coherence sector. The space of trilinear maps $T_{SBF} : H_S \times H_B \times H_F \to \mathbb{C}$ has dimension $6^3 = 216$. We must show these are not all representable within the rank-18 space without an ordering parameter.

The key observation is that the trilinear space $H_S \otimes H_B \otimes H_F$ has no canonical decomposition into symmetric and antisymmetric parts without a choice of ordering on $\{S, B, F\}$. Specifically, the symmetric group $\mathfrak{S}_3$ acts on $H_S \otimes H_B \otimes H_F$ by permuting the three factors, but the action is non-trivial (the three bearings are distinguishable). The space decomposes under $\mathfrak{S}_3$ into irreducible representations:

$$H_S \otimes H_B \otimes H_F \cong V^{(3)} \oplus V^{(2,1)} \oplus V^{(1,1,1)} \tag{11}$$

where $V^{(3)}$ is the fully symmetric part, $V^{(1,1,1)}$ the fully antisymmetric part, and $V^{(2,1)}$ the mixed-symmetry part. Critically, the mixed-symmetry sector $V^{(2,1)}$ requires a choice of *which two bearings to symmetrise* to be well-defined—this choice is precisely the ordering scalar.

Without the ordering scalar, the mixed-symmetry sector of $H_S \otimes H_B \otimes H_F$ is not canonically representable: different orderings produce different, non-isomorphic decompositions. A rank-18 representation that attempts to track all of $H_S \otimes H_B \otimes H_F$ without fixing an ordering cannot distinguish between these decompositions and therefore cannot faithfully represent the mixed-symmetry invariants. The ordering parameter is the minimal repair: it resolves the $\mathfrak{S}_3$ ambiguity by selecting a canonical decomposition. □

The ordering scalar that appears at rank 18 is recognisable in physical models as the time parameter t in time-ordered products, or as the renormalisation scale μ in Wilsonian effective theory. Its appearance is not a fundamental physical choice; it is a symptom of representational insufficiency at rank 18.

8 Closure Constraints

We now formalise the three invariance constraints that a faithful participatory representation must satisfy. Each constraint captures a structural property that cannot be compromised without introducing repair terms.

8.1 Persistence

Definition 8.1 (Persistence)**.** A representation satisfies **Persistence** if its relational invariants are preserved under all admissible internal transformations without scalar compensation:

$$\mathcal{I}(T(e)) = \mathcal{I}(e) \qquad \text{for all admissible } T. \tag{12}$$

Failure of persistence appears as running couplings or scale-dependent effective parameters.

8.2 Generativity

Definition 8.2 (Generativity)**.** A representation satisfies **Generativity** if the interaction product of any two tokens produces a token whose invariants are not expressible as a function of the invariants of the inputs:

$$\mathcal{I}(e_i \circ e_j) \neq f(\mathcal{I}(e_i), \mathcal{I}(e_j)) \qquad \text{for any function } f \text{ and for } e_i \neq e_j. \tag{13}$$

Equivalently, $e_i \circ e_j$ generates a relational invariant that is *linearly independent* of the invariants of e_i and e_j individually. Failure of generativity means the product carries no new information beyond what was already present in the factors; the algebra is non-productive.

This formulation is stronger than the bare non-idempotency condition $e_i \circ e_j \notin \{e_i, e_j\}$ and is structurally load-bearing: it requires the product to generate a new degree of freedom in the invariant space, which is precisely the condition that prevents the Generativity constraint from being vacuously satisfied by any non-trivial algebra. Under the Non-Retraction Condition, this new invariant must be tracked by a dedicated representational degree of freedom; it cannot be identified with any existing invariant without retraction.

8.3 Orthogonality

Definition 8.3 (Orthogonality)**.** A representation satisfies **Orthogonality** if relational invariants are frame-independent: no privileged coordinate choice is required to evaluate them. Failure of orthogonality appears as coordinate-dependent effective parameters or calibration-sensitive measurements.

9 Minimal Closure at Rank 27: The Martin Postulate

The three structural bearings and three closure constraints interact as follows. Each constraint is not a global property of the representation but must hold *across each bearing independently*: Persistence, Generativity, and Orthogonality must each be satisfied within H_S, within H_B, and within H_F—and furthermore in the cross-bearing interaction structure. This means each bearing must carry sufficient harmonic content to support all three constraints without projecting onto the others.

Before the main theorem we establish a key lemma that closes a potential circularity in the lower-bound argument.

Lemma 9.1 (Per-Pair Harmonic Independence)**.** *Each of the nine constraint-bearing pairs $(\mathcal{K}, X)$ where $\mathcal{K} \in \{$Persistence, Generativity, Orthogonality$\}$ and $X \in \{S, B, F\}$ requires at least 3 independent harmonic degrees of freedom to avoid functional collapse within that sector.*

Proof. Fix a constraint $\mathcal{K}$ and a bearing X. The sector $(\mathcal{K}, X)$ tracks the invariants of $\mathcal{K}$ as they manifest within bearing H_X. Specifically:

- *Persistence* within H_X requires that the three intra-bearing invariants of H_X are preserved under all admissible transformations. By Theorem 4.1, three independent invariants require three harmonic degrees within H_X.

- *Generativity* within H_X requires that the product of any two tokens in H_X generates a new linearly independent invariant (Definition 8.2). The new invariant lies in $H_X \otimes H_X$ (degree-2 sector relative to H_X) and is linearly independent of the degree-1 invariants by the tensor algebra grading. Tracking both degree-1 and degree-2 invariants within the sector requires $\dim H_X \geq 3$.

- *Orthogonality* within H_X requires that invariants be evaluable without a privileged frame, which by Section 5.2 requires harmonic structure—i.e., a $U(1)$ gauge freedom. The minimal harmonic space supporting three independent invariants has $\dim H_X = 3$.

In each case the sector $(\mathcal{K}, X)$ requires at least 3 independent harmonic degrees. Since the constraint imposes independent requirements on each bearing, and bearings are orthogonal subspaces (by Proposition 5.2 and the Orthogonality constraint), the degrees required by different sectors within the same bearing cannot be shared: sharing would imply that one degree of freedom simultaneously tracks two independent invariants from different constraint sectors, which violates linear independence. □

Theorem 9.2 (Martin Postulate)**.** *Under the Non-Retraction Condition, with three structural bearings and three closure constraints (Persistence, Generativity, Orthogonality), and in the absence of scalar normalisation or privileged ordering, the minimal intrinsic representational rank is* ***27****.*

Proof. Lower bound. By Lemma 6.1, rank 9 fails under adversarial compression (Definition 1.1) with respect to the combined set of intra-bearing and cross-bearing invariants. By Lemma 7.1, rank 18 fails under adversarial compression with respect to triple-coherence invariants. It remains to show that all ranks r with $18 < r < 27$ also fail.

By Lemma 9.1, each of the 9 constraint-bearing pairs $(\mathcal{K}, X)$ requires at least 3 linearly independent harmonic degrees. Since the 9 sectors involve orthogonal subspaces of distinct tensor grades, their degree requirements are non-overlapping: no single representational degree of freedom can simultaneously satisfy the independence conditions for two distinct sectors. Therefore the total degrees required is at least $9 \times 3 = 27$. For any $r < 27$, at least one sector has fewer than 3 degrees, inducing functional collapse in that sector by Lemma 9.1.

Upper bound. A rank-27 representation assigns exactly 3 harmonic degrees to each of the 9 constraint-bearing sectors. All intra-sector invariants are tracked without identification. Cross-sector invariants are spanned by the full product structure of the 27-dimensional space. The trilinear sector $H_S \otimes H_B \otimes H_F$ is now of dimension $3 \times 3 \times 3 = 27$, which matches the total rank exactly: the rank-27 space is isomorphic to $H_S \otimes H_B \otimes H_F$ as a vector space, and therefore spans the full trilinear invariant space without residual scalar degrees of freedom. Thus rank 27 is sufficient.

Therefore 27 is both necessary and sufficient under the stated conditions. □

Remark 9.3. The number $27 = 3^3$ is not numerologically significant in isolation. It is the product of three independent triadic requirements: harmonic non-degeneracy (3), structural bearing count (3), and closure constraint count (3). Changing any of these inputs changes the minimal rank accordingly.

9.1 The Ever-Jittering Fulcrum: 27 as Axial Kernel, Not Fixed Point

Theorem 9.2 establishes 27 as a sharp lower bound under the stated constraints. The theorem itself is falsifiable in the standard sense: a representation of rank less than 27 satisfying all three constraints without scalar repair would directly refute it (see Section 14, condition F1).

A distinct but related question concerns *where in a given physical or economic system* the rank-27 closure is operative. The companion paper [1] identifies this as the problem of the **Ever-Jittering Fulcrum**: the extremum of the action functional is never exactly achieved,

$$\delta W \approx 0 \qquad \text{(approximate, never exact)}, \tag{14}$$

so position—and, analogously, representational rank—is perpetually regenerated rather than statically occupied. Applied to rank: a real system does not sit at exactly rank-27 closure but orbits it, approaching from above (partially over-determined) and below (partially under-determined) as its internal degrees of freedom are engaged and suppressed.

This dynamic character has two distinct implications that must be kept separate:

(i) The theorem's validity is not affected. Theorem 9.2 establishes the minimum rank for *exact* closure—the rank below which at least one invariant must be retracted. This is a structural result about the representational algebra and is independent of any particular system's dynamical behaviour. The Ever-Jittering Fulcrum does not make the theorem unfalsifiable; it describes the dynamical regime in which a real system operates relative to the theoretical minimum.

(ii) Empirical localisation is necessary but distinct from theoretical validation. The theoretical result tells us that rank-27 closure is the minimum; it does not tell us the *scale parameter* $\Lambda_{\mathcal{C}}$ at which this minimum manifests in any given system. This is analogous to quantum mechanics predicting discrete energy levels without fixing Planck's constant $\hbar$: the structure is theoretical, but the scale must be measured.

The Innovation Alpha indexes reported in [1] provide this localisation in the economic domain. The relevant claim is not that 27 dimensions outperform 26 in patent analysis—no such pairwise comparison is asserted—but that the *full rank-27 representation* avoids the structural failure modes identified in Lemmas 6.1 and 7.1: cross-bearing invariant collapse and triple-coherence ordering failure. The 5–9% annual excess returns are evidence that these failure modes are operationally significant at market scale—that collapsing the patent network topology below rank-27 closure costs measurable information. Similarly, the physiological coherence boundary measurements ($R^2 > 0.82$, $N = 47$) localise the axial kernel in the autonomic nervous system: the convergence of HRV, EEG phase-lock, and parasympathetic amplitude at the boundary state is the physiological signature of rank-27 closure.

Neither empirical result is required to *validate* the theorem. Both are required to *localise* the theorem's consequences in specific physical systems—a step that is empirically necessary but logically independent of the representational-algebraic argument.

10 Renormalisation as Retraction Accounting

Wilson's renormalisation group programme [3] proceeds by integrating out high-energy modes, producing an effective action

$$S_{\text{eff}} = S + \Delta S, \tag{15}$$

where ΔS is the correction arising from projected modes. In the framework of the present paper, ΔS is repair structure: it compensates for the retraction induced by the mode projection π. The counterterms in ΔS carry the invariants that the projected-out modes had been tracking.

A clarification is essential to prevent misreading. Running couplings in *full* quantum field theory—e.g., the running of the QED fine-structure constant α in the complete theory—arise from ultraviolet structure and are present even without any explicit mode elimination. The present argument does *not* claim that all coupling running is a projection artefact.

The claim is specifically and only about the *Wilsonian* context: when one explicitly performs a projection π that eliminates modes above a cutoff Λ_{UV}, the dependence of the resulting effective couplings on Λ_{UV} is a direct consequence of that projection. Polchinski's exact renormalisation group [4] makes this explicit—the flow equation

$$\Lambda \frac{\partial S_\Lambda}{\partial \Lambda} = (\text{functional of } S_\Lambda) \tag{16}$$

is precisely the differential equation governing how scalar repair accumulates as the representation is compressed below its faithful rank by successively lowering Λ.

This scoped claim is sufficient for all engineering corollaries in Section 13, which concern systems where mode elimination (circuit QED) or spatial averaging (metamaterials) is explicitly performed. It makes no claim about UV running in the fundamental theory.

11 Scalar Purgatory

Definition 11.1 (Scalar Purgatory)**. Scalar Purgatory** is the regime in which closure is maintained through scalar repair terms compensating for representational collapse below the minimal faithful rank.

Scalar purgatory is characterised by proliferation of effective parameters whose values cannot be predicted from first principles but must be measured or calibrated. The parameters are not arbitrary—each one corresponds to a specific collapsed invariant—but their presence reflects the structural cost of sub-minimal representation. Canonical examples include:

- Running couplings $\alpha_s(\mu)$, $g(\mu)$ in quantum chromodynamics and electroweak theory.
- Effective masses and Lamb shifts in circuit QED after cavity mode elimination.
- Calibration parameters in metamaterial effective medium theory.
- The time parameter t in time-ordered perturbation theory.

Each parameter listed above is, in the Non-Retraction framework, a ghost of a collapsed degree of freedom. Its value can in principle be computed from a higher-rank faithful representation, but in practice it is treated as an empirical input.

12 Relation to Relational Physics

Rovelli's relational quantum mechanics [5] holds that quantum states are defined only relative to observers, with no observer-independent wave function. This is a relational position at the level of physics, but it does not engage the question of representational rank. The present framework is complementary: it asks, given that all structure is relational, what is the minimal representation that can carry the relevant relational invariants faithfully?

Algebraic quantum field theory in the Haag–Kastler tradition [6] organises physics around algebras of observables associated to spacetime regions, prioritising the algebraic structure over Hilbert space representations. The Non-Retraction Condition is consistent with this prioritisation: it asks that the algebra be represented faithfully, without collapsing algebraic distinctions. The minimal rank result can be stated in that language as a theorem about the minimal faithful representation of the event algebra $\mathscr{E}$ under the three stated constraints.

Neither framework derives a minimal representational rank from a non-retraction constraint. The contribution of the present work is to make the constraint explicit, to show that three progressively adversarial compressions (ranks 2, 9, and 18) each fail, and to establish 27 as the minimal rank at which failure ceases.

13 Engineering Corollaries

The structural argument for rank-27 closure has concrete implications for physical systems in which boundary participation is operationally significant.

13.1 Circuit QED

In circuit quantum electrodynamics the standard Hamiltonian takes the form

$$H = H_q + H_m + H_{\text{int}}, \tag{17}$$

where H_q is the qubit (system) Hamiltonian, H_m the microwave cavity (mode) Hamiltonian, and H_{int} the coupling. Standard practice eliminates cavity modes via adiabatic elimination or Schrieffer–Wolff transformation, producing Lamb shifts and renormalised dispersive couplings.

In the Non-Retraction framework, the coupling boundary B is the interface between qubit and cavity. Standard mode elimination projects out degrees of freedom in H_B, inducing retraction of the cross-bearing structure. The Lamb shift is the repair term compensating for this retraction. *Prediction*: an encoding that retains explicit harmonic boundary degrees of freedom (rather than eliminating them) will exhibit reduced Lamb shift drift and greater parameter stability across detuning regimes.

13.2 Photonic and Metamaterial Systems

Effective medium theory replaces heterogeneous boundary structure with scalar parameters ε_{eff}, μ_{eff}. These are obtained by spatial averaging, which collapses the boundary participation of the individual scatterers. In Non-Retraction terms, the averaging is a projection that identifies distinct boundary tokens $b_i \neq b_j$ by mapping them to the same effective parameter value.

Prediction: replacing scalar effective parameters with harmonic boundary encodings that represent the boundary structure at rank sufficient to avoid identification should reduce the frequency dependence and spatial dispersion of the effective parameters. This is consistent with observations that homogenisation breaks down near resonances, where boundary participation is maximally distinct.

13.3 Boundary-Sensitive Electromagnetic Systems

More broadly, any electromagnetic system in which boundary conditions are operationally significant—antenna near-field coupling, photonic crystal defect modes, impedance-matched absorbers—involves a coupling boundary bearing B that participates non-trivially in the system dynamics. Standard computational approaches (finite-element methods, mode matching) typically project boundary degrees of freedom onto scalar boundary conditions. The Non-Retraction framework predicts that this projection will introduce calibration sensitivity—a need to re-tune boundary parameters when system geometry or frequency changes—in proportion to the degree to which boundary participation is collapsed.

14 Falsifiability and Scope

The Martin Postulate consists of two separable claims that admit distinct falsification conditions.

Claim A: The Structural Theorem

Theorem 9.2 asserts that no representation of rank less than 27 can satisfy the Non-Retraction Condition jointly with all three closure constraints without scalar repair. This claim is falsifiable in the following precise senses:

F1. Exhibit a representation of rank $r < 27$ that satisfies all three closure constraints (Persistence, Generativity, Orthogonality as formalised in Section 8) and the Non-Retraction Condition (Definition 3.1) simultaneously, with no scalar repair terms. This would directly refute Theorem 9.2.

F2. Show that at least one of the three structural bearings (System, Coupling Boundary, Delimiting Frame) is reducible to a combination of the other two without loss of the relational invariants of Proposition 5.2. This would reduce the bearing factor from 3 to 2, changing the minimal rank from 27 to 12.

F3. Show that the Generativity constraint (Definition 8.2) does not generate new linearly independent invariants—i.e., that $\mathcal{I}(e_i \circ e_j)$ is always expressible as $f(\mathcal{I}(e_i), \mathcal{I}(e_j))$ for some function f. This would reduce the constraint factor from 3 to 2, giving a minimal rank of 18.

F4. Show that the linear independence argument in Lemma 9.1 fails—that two constraint-bearing sectors can share degrees of freedom without retraction. This would break the $9 \times 3 = 27$ counting argument.

Claim B: The Localization Assertion

The Ever-Jittering Fulcrum discussion in Section 9.1 makes a further, empirically-loaded claim: that the rank-27 closure identified by Theorem 9.2 is *operative in specific real systems* (patent networks, the autonomic nervous system). This claim is *independent* of Claim A and has its own falsification conditions:

F5. Demonstrate that a representation of dimension substantially less than 27 captures the same predictive performance in patent network analysis—specifically, that the structural failure modes of Lemmas 6.1 and 7.1 (cross-bearing collapse, ordering parameter proliferation) are absent at lower rank. This would undermine the application to Innovation Alpha without affecting Theorem 9.2.

F6. Show that the HRV/EEG phase-lock convergence described in [1] can be fully explained by a model with fewer than 27 effective degrees of freedom—i.e., that no invariants are retracted at the boundary state.

The Ever-Jittering Fulcrum does *not* render Claim A unfalsifiable. Claim A is falsified by constructive counterexample (conditions F1–F4 above). The dynamic character of real systems near the rank-27 kernel is a separate matter: it explains why any given measurement will see the system *near* 27 rather than *at* exactly 27, but the theorem's validity does not depend on any real system sitting exactly at the bound.

The postulate does *not* assert that all physical systems must be modelled at rank 27, or that projection is never appropriate. It claims only that projection below rank 27 has a structural cost—quantifiable as scalar repair—and that this cost is zero at rank 27 under the declared constraints.

15 Conclusion

The Martin Postulate emerges not as assertion but as the outcome of a sequence of adversarial compression arguments, each closed by an explicit linear independence condition. The argument proceeds in five stages: (1) dyadic representations are shown insufficient for three co-primitive aspects by a direct cardinality argument; (2) rank-9 failure is established by showing that intra-bearing and cross-bearing invariants are linearly independent elements of distinct tensor grades, with their combined count exceeding the available degrees of freedom; (3) rank-18 failure is established by the $\mathfrak{S}_3$ symmetry argument: without an ordering scalar the mixed-symmetry sector of the trilinear invariant space is not canonically representable; (4) the per-pair independence lemma (Lemma 9.1) closes the lower-bound proof by showing that no two constraint-bearing sectors can share harmonic degrees without retraction; (5) the Ever-Jittering Fulcrum Principle is shown to concern the *localisation* of the kernel in real systems, not the validity of the theorem itself—the two claims are separated and each is given its own falsification conditions.

The structural correspondence with 27-Dimensional Triangulation is complete and non-accidental. The three structural bearings (System, Coupling Boundary, Delimiting Frame) are the three triangulation domains of 27DT (Observed Phenomenon, Emergent Context, Observer Delimitations), each irreducible by the argument of Section 5. The three closure constraints (Persistence, Generativity, Orthogonality) are the three Source Principles of 27DT ($\mathcal{P}$, $\mathcal{G}$, $\mathcal{O}$), with Generativity now formulated as a structural independence condition rather than a bare non-idempotency requirement. Both frameworks arrive at $27 = 3^3$ by independent routes; the convergence is the structural argument.

The practical upshot is a diagnostic framework for scalar proliferation in physical models, with a scoped empirical commitment. When a Wilsonian effective model requires calibration parameters, running couplings, or ordering scalars unpredictable from first principles, the Non-Retraction framework identifies these as consequences of projection below minimal faithful rank. The engineering corollaries give three testable predictions: reduced Lamb shift drift in circuit QED under boundary-preserving encodings, reduced dispersion in metamaterial effective parameters under higher-rank boundary representations, and systematic calibration sensitivity in boundary-significant electromagnetic systems proportional to the degree of boundary collapse. These are the empirical content of the theorem in the physical domain.

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